

Drag effects in the system of electrons and microcavity polaritons

Oleg L. Berman¹, Roman Ya. Kezerashvili^{1,2}, and Yurii E. Lozovik^{3,4}

¹*Physics Department, New York City College of Technology, The City University of New York, Brooklyn, NY 11201, USA*

²*The Graduate School and University Center, The City University of New York, New York, NY 10016, USA*

³*Institute of Spectroscopy, Russian Academy of Sciences, 142190 Troitsk, Moscow Region, Russia*

⁴*Moscow Institute of Physics and Technology (State University), 141700, Dolgoprudny, Russia*

(Dated: August 31, 2010)

The theory of the drag effects in the system of spatially separated electrons and excitons in coupled quantum wells (QW) embedded in an optical microcavity is developed. It is shown that at low temperature an electron current induces the (normal component) polariton flow, therefore, a transport of photons along the cavity. However, the electron current dragged by the polariton flow is strongly suppressed below polariton superfluid transition temperature and hence, the strong suppression of the induced electron current indicates the superfluidity of polaritons. Therefore, the transport properties of polaritons can be investigated by measuring the current or voltage in the electron subsystem. At high temperatures we study the exciton-electron drag effects. At high temperatures regime, from one hand, the existence of the electric current in an electron QW induces the exciton flow in the other QW, from the other hand, the electron current in one QW induces the exciton flow in the other QW via the drag of excitons by the electrons. The drag coefficients for the polariton-electron systems are calculated and analyzed. We discuss the possible experimental observation of the drag effects in the system of electrons and microcavity polaritons, that also allow to observe the cavity polaritons superfluidity.

PACS numbers: 71.36.+c, 71.35.-y, 03.75.Kk, 73.21.Fg

I. INTRODUCTION

The study of the drag effects in a two-layer system of electrons and holes, and electrons and excitons has long history. The drag effects in a two-layer system of spatially separated electrons and holes were predicted theoretically, and their influence on phase transitions to a superfluid excitonic phase investigated in Ref. [1]. Later, Pogrebinskii in Ref. [2] discussed the drag of electrons by electrons in a semiconductor–insulator–semiconductor structure. Price proposed a practical method for observing the drag effect in heterostructures [3]. Subsequently, the drag effect was explored in a number of theoretical and experimental studies [4–13], where various physical realizations of the drag effect were discussed in one-dimensional, two-dimensional, and three-dimensional systems. The prediction was that for two conducting layers separated by an insulator there will be a drag of carriers in one film due to the direct Coulomb interaction with the carriers in the other film.

A theory of transresistivity coefficients, which relate electric fields in one layer to currents flowing in the opposite layer, for the case of an electron-hole double-layer system with a superfluid electron-hole condensate was presented in Ref. [12]. Such electron-hole system can be realized in GaAs/AlGaAs coupled quantum wells (CQWs). The measurement of these transport coefficients could provide an unambiguous experimental indication of the existence of the condensate. When no condensate is present, the transresistivity is typically several orders of magnitude smaller than the isolated layer resistivity. According to Ref. [12], the transresistivity jumps to a value comparable to the isolated layer resistivity as soon as the condensate forms, it continues to increase with decreasing temperature T , and it diverges as $T \rightarrow 0$. Employing diagrammatic perturbation theory, the charge Coulomb drag resistivity and spin Coulomb drag resistivity were calculated in the presence of Rashba spin-orbit coupling in Ref. [14].

For the past decade there was great amount of various interesting experiments on drag effects performed in the different groups. The measurements in a perpendicular magnetic field of the frictional drag between two closely spaced, but electrically isolated, two-dimensional electron gases were presented by Pepper's group [15]. The drag measurements on dilute double layer two-dimensional hole systems have been performed in Tsui's group [16]. The formation of the superfluid exciton Bose condensate at low temperatures has been experimentally studied by performing magnetotransport and drag measurements on a quasi-Corbino two-dimensional electron bilayer at a total filling factor of 1 by von Klitzing's group [17]. Electron-hole bilayers are expected to make a transition from a pair of weakly coupled two-dimensional systems to a strongly coupled exciton system as the barrier between the layers is reduced. The recent measurements done by Lilly's group [18] for Coulomb drag in the exciton regime in electron-hole bilayers demonstrated that there is an increase in the drag resistance as the temperature is reduced when a current is driven in the electron layer and voltage measured in the hole layer. These results indicate the onset of strong coupling possibly

due to exciton formation or phenomena related to exciton condensation.

The kinetic properties of a two-layer system of electrons and excitons at temperatures above the temperature T_c of the Kosterlitz–Thouless transition [19], at which there is no local condensate or superfluidity in the exciton system, were considered in Ref. [20]. The kinetic properties of a two-layer system of electrons and excitons at temperatures below the temperature T_c under conditions allowing the existence of a Bose condensate of excitons have been studied in Ref. [21]. The corresponding calculations have been performed for two-dimensional systems of spatially separated electrons and excitons. The effect of spatially separated electron–exciton drag in a double layer system was studied in the Debye–Hückel approximation taking into account screening of the interlayer electron–exciton interaction [22].

Recently Bose coherent effects of two-dimensional (2D) excitonic polaritons in a quantum well embedded in an optical microcavity have been the subject of theoretical and experimental studies [23–25]. To obtain polaritons, two mirrors placed opposite each other form a microcavity, and quantum wells are embedded within the cavity at the antinodes of the confined optical mode. The resonant exciton-photon interaction results in the Rabi splitting of the excitation spectrum. Two polariton branches appear in the spectrum due to the resonant exciton-photon coupling. The lower polariton branch of the spectrum has a minimum at zero momentum. The effective mass of the lower polariton is extremely small, and lies in the range $10^{-5} - 10^{-4}$ of the free electron mass. These lower polaritons form a 2D weakly interacting Bose gas. The extremely light mass of these bosonic quasiparticles at experimentally achievable excitonic densities, results in a relatively high critical temperature for superfluidity, of 100 K or even higher, because the 2D thermal de Broglie wavelength is inversely proportional to the mass of the quasiparticle.

While at finite temperatures there is no true BEC in any infinite untrapped 2D system, a true 2D BEC quantum phase transition can be obtained in the presence of a confining potential [26, 27]. The essential experimental progress was achieved in experimental studies of exciton polaritons in the system of a QW embedded in optical microcavity [28–30]. Recently, the polaritons in a harmonic potential trap have been studied experimentally in a GaAs/AlAs quantum well embedded in a GaAs/AlGaAs microcavity [31]. In this trap, the exciton energy is shifted using a stress-induced band-gap. In this system, evidence for the BEC of polaritons in a quantum well has been observed [32, 33]. The theory of the BEC and superfluidity of excitonic polaritons in a quantum well in a parabolic trap has been developed in Ref. [34]. The Bose condensation of polaritons is caused by their bosonic character [32, 34, 35]. The BEC and superfluidity of cavity polaritons in a QW without a trap were considered in [37–41].

The investigation of the kinetic properties of a system of spatially separated polaritons and electrons based on drag effects can provide additional information regarding the phase state of the polariton subsystem and phase transitions in it. The phase state of the polariton subsystem can be analyzed by performing a simple study of the response of the electron subsystem. In other words, the transport properties of polaritons and their changes upon phase transitions can be investigated by measuring the current or voltage in the electron subsystem. Another property of systems of spatially separated interacting quasiparticles is the possibility of controlling the motion of the quasiparticles of one subsystem by altering the parameters of state of the quasiparticles in the other subsystem (for example, controlling the motion of electrons using a flow of polaritons). While the drag effect in various electron–hole systems were considered in many interesting papers cited above, we are lacked of such research for a polariton-electron system. The drag effect in the polariton-electron system embedded in an optical microcavity was predicted just recently [36]. In this Paper we develop the theory of the drag effects in the system of spatially separated electrons and excitons in coupled quantum wells embedded in an optical microcavity.

The Paper is organized in the following way. In Sec. II we present the interaction Hamiltonian between the spatially separated polaritons and electrons. In Sec. III we study transport relaxation time of the quasiparticle excitations and polaritons. In Sec. IV we calculate the drag coefficients corresponding to the drag of the quasiparticles in the polariton subsystem by the electron current and the drag of the electrons by the quasiparticles in the polariton subsystem. The study of the drag effects in the exciton-electron system at high temperatures is presented in Sec. V. In Sec. VI we propose the experiments to observe the drag effects. Finally, the discussion of the results and conclusions follow in Sec. VII.

II. INTERACTION OF POLARITONS WITH ELECTRONS

We consider two neighboring quantum wells embedded in an optical microcavity: the first QW is occupied by 2D electron gas (2DEG) and the second QW is occupied by the excitons created by the laser pumping.

At low temperature $k_B T \ll \hbar \Omega_R$, where $\hbar \Omega_R$ is Rabi splitting, and k_B is Boltzmann constant, and at the resonance of excitons with cavity photons, the excitons are entangled with the cavity photons and form the exciton polaritons. Here we omit weak effects of direct nonresonant interaction of photons with 2D electron gas which will be considered elsewhere. By focusing laser pumping in some region of the cavity the gradient of excitons and exciton polaritons densities can be generated. These gradients induce the polariton and exciton flows, and in a turn the normal component of moving excitons drags the electrons in the neighboring QW due to the electron-exciton interaction. So the electric

current would be generated by the flow of the normal component of exciton polaritons.

In another scenario by applying electric voltage in the QW with 2DEG the electronic current is induced, and this current drags the normal component of excitons in the neighboring QW. The excitons are entangled with the cavity photons. So the cavity photons can be also dragged and, thus, controlled by the electric voltage. Besides, possible applications of the control of photons and (or) excitons by the exciton-electron drag can be used for study of the properties and phases in the polariton and exciton system, particularly, the superfluidity of the system.

From the other hand, at high temperature, $k_B T \gtrsim \hbar \Omega_R$, the polariton states are occupied mainly far from the polariton resonance, and in these states exciton-photon entanglement is negligible. Thus, at high temperatures only the exciton-electron drag is essential, and the exciton flow can induce the electron current as well as the electron current can produce the exciton flow. The various drag effects in an optical microcavity considered in this Paper are schematically shown in Fig. 1.

We consider the system of two parallel quantum wells embedded into a semiconductor microcavity. One quantum well is filled by two-dimensional exciton polaritons formed by direct excitons and microcavity photons. We assume that the exciton system is dilute and weakly interacting system, satisfying to the following condition $n_{ex} a_0^2 \ll 1$, where n_{ex} is the 2D exciton density, $a_0 = \epsilon \hbar^2 / (2 \mu_0 e^2)$ is the 2D exciton Bohr radius, $\mu_0 = m_e m_h / (m_e + m_h)$ is the reduced exciton mass, m_e and m_h are effective masses of an electron and a hole, respectively, ϵ is the dielectric constant in a quantum well, e is the charge of an electron. The condition $n_{ex} a_0^2 \ll 1$ holds for the excitons at the exciton densities up to $n_{ex} \approx 10^{12} \text{ cm}^{-2}$, since in GaAs/GaAsAl quantum well the exciton Bohr radius is in the order of $a_0 \sim 10 - 50 \text{ \AA}$. It is obvious to conclude that the system of polaritons is also weakly interacting system, since the system of excitons, forming the polaritons, is dilute. The other quantum well parallel to the quantum well with excitons is filled by the 2D electron gas (2DEG). The polariton-electron drag effects are caused by the polariton-electron interaction.

Let us mention that the second QW filled by the 2DEG can also possess excitons, because the excitonic population can be driven by the microcavity photons. Therefore, in this system of the coupled QWs embedded in an optical microcavity, the formation of polaritons, whose exciton components are shared on both the QWs, can occur. However, we do not consider the excitons formation in the QW filled by the 2DEG, since we suggest constructing this 2DEG QW so that the excitons in this QW are not in the resonance with the microcavity photons and the excitons in the other, “excitonic” QW. This can be achieved by the fact that the two QWs embedded in an optical microcavity are assumed to have different chemical compositions or different widths.

The potential energy of the pair attraction between the exciton and electron placed in two parallel quantum wells with the spatial separation D is given by [20, 21]

$$W(r, D) = -\frac{21}{32} \frac{e^2 a_0^3}{\epsilon (r^2 + D^2)^2}, \quad (1)$$

where r is the distance between the exciton and electron along the plane. The 2D Fourier image of $W(r, D)$ is

$$W(q, D) = -\frac{21\pi}{32} \frac{e^2 a_0^3}{\epsilon} \frac{q}{\hbar D} K_1(qD/\hbar), \quad (2)$$

where $K_1(qD)$ is the modified Bessel function of the second kind.

In a many-particle system of excitons and electrons the bare pair interaction $W(q, D)$ should be replaced by the effective interaction $W_{eff}(q, D)$ corresponding to the exciton-electron interaction screened by the electron-electron interactions in 2DEG. In the system of dilute excitons in one quantum well and 2DEG in the other quantum well $W_{eff}(q, D)$ is given by [21]

$$W_{eff}(q, D) = -\frac{W(q, D)}{1 - \Pi_e(q) V_e(q)}, \quad (3)$$

where $V_e(q) = 2\pi e^2 \hbar / (\epsilon q)$ is the 2D Fourier image of the pair electron-electron Coulomb attraction, and $\Pi_e(q)$ is the 2D polarization function for electrons. We consider the limit of the very dense 2DEG, when the following condition is valid: $n_{el} a_e^2 \gg 1$, where n_{el} is the 2D density of the electrons and $a_e = \epsilon \hbar^2 / (2 m_e e^2)$ is the 2D electron Bohr radius. In the limit of very dense 2DEG in the Thomas-Fermi approximation $\Pi_e(q)$ is $\Pi_e(q) = m_e / (\pi \hbar^2)$ [42], and, therefore, $\Pi_e(q) V_e(q) = (a_e q / \hbar)^{-1}$.

At low exciton density, $n_{ex} a_0^2 \ll 1$, the term of the Hamiltonian \hat{H}_{ex-el} corresponding to the exciton-electron interaction has the form [21]

$$\hat{H}_{ex-el} = \frac{1}{A} \sum_{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}'_1, \mathbf{p}'_2} W_{eff}(|\mathbf{p}_1 - \mathbf{p}'_1|, D) \hat{a}_{\mathbf{p}_1}^\dagger \hat{c}_{\mathbf{p}'_2}^\dagger \hat{c}_{\mathbf{p}_2} \hat{a}_{\mathbf{p}'_1}, \quad (4)$$

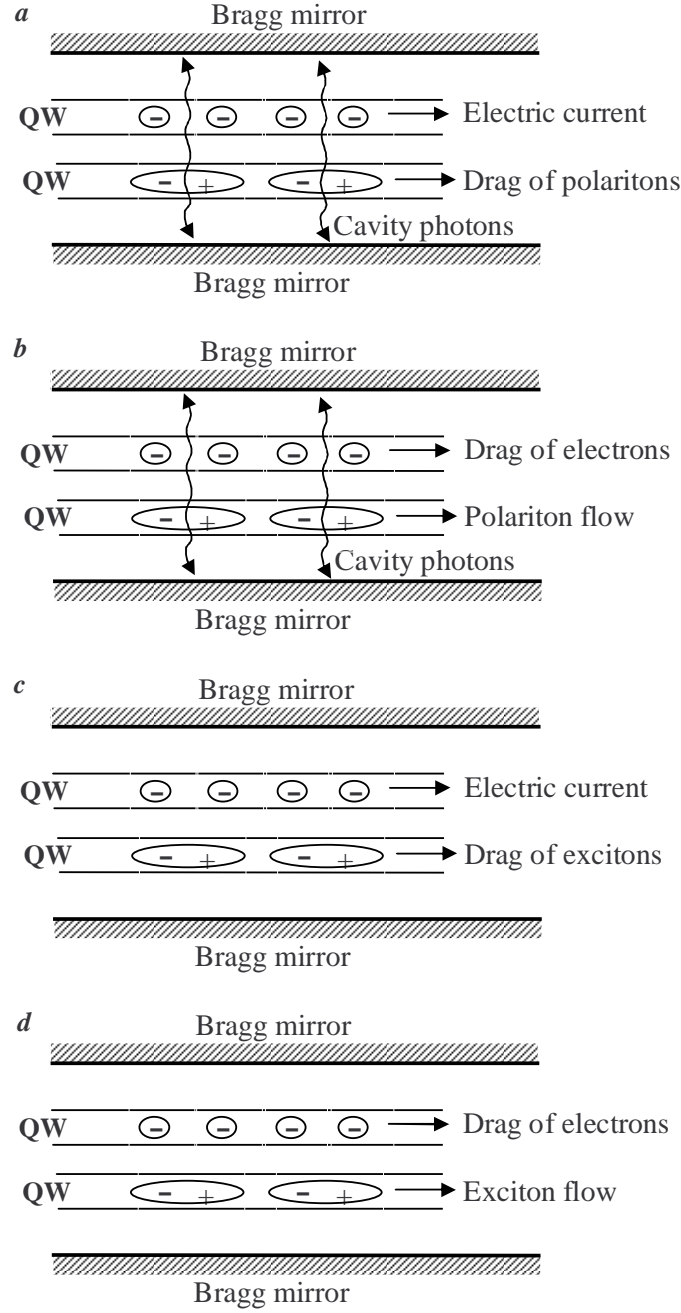


FIG. 1: The schematic diagram for the drag effects in the CQWs embedded in an optical microcavity. a. The quasiparticles in the cavity polariton subsystem are dragged by the electron current at low temperatures. b. The electrons are dragged by the flow of the quasiparticles in the cavity polariton subsystem at low temperatures. c. The excitons are dragged by the electron current at the high temperatures. d. The electrons are dragged by the exciton flow at high temperatures.

where $\hat{a}_{\mathbf{p}}^{\dagger}$ and $\hat{a}_{\mathbf{p}}$ are the exciton Bose creation and annihilation operators, respectively, $\hat{c}_{\mathbf{p}}^{\dagger}$ and $\hat{c}_{\mathbf{p}}$ are the electron Fermi creation and annihilation operators, respectively, and A is the macroscopic quantization area of the system. Below for the derivation of the drag coefficients we need the Hamiltonian of the exciton-electron interaction in the representation of the operators of the quasiparticles in the polariton subsystem. The details for the derivation of the Hamiltonian of the exciton-electron interaction in the representation of the quasiparticle operators are presented in Appendix A.

III. TRANSPORT RELAXATION TIME OF THE QUASIPARTICLE EXCITATIONS AND POLARITONS

In order to calculate polariton-electron and exciton-electron drag coefficients we use the kinetic equations in Sec. IV, which include the transport relaxation time $\tau_1(p)$ of the quasiparticle excitations corresponding to the scattering on the impurities enters. In this Section we obtain the transport relaxation time to the scattering of the quasiparticles on the impurities. The Hamiltonian of elastic interactions of excitons with impurities is given by

$$\hat{H}_1 = \frac{1}{A} \sum_{\mathbf{p}, \mathbf{p}'} V(\mathbf{p}, \mathbf{p}') \hat{a}_{\mathbf{p}}^\dagger \hat{a}_{\mathbf{p}'} , \quad (5)$$

where $V(\mathbf{p}, \mathbf{p}')$ is the matrix element for the interactions of an exciton with impurities. Replacing the exciton operators by the polariton operators according to Eqs. (A1) and the polariton operators by the operators for quasiparticle excitations defined by Eqs. (A10), and keeping only the terms which satisfy the requirement of elastic collisions, we obtain

$$\hat{H}_1 = \frac{1}{A} \sum_{\mathbf{p}, \mathbf{p}'} V(\mathbf{p}, \mathbf{p}') \sigma(p, p') \hat{b}_{\mathbf{p}}^\dagger \hat{b}_{\mathbf{p}'} , \quad (6)$$

Using Hamiltonian (6) and applying Fermi's golden rule, we obtain for the reciprocal of the transport relaxation time $\tau_1(p)$

$$\frac{1}{\tau_1(p)} = \frac{2\pi}{\hbar} \int |V(\mathbf{p}, \mathbf{p}') \sigma(p, p')|^2 \delta(\varepsilon_1(p) - \varepsilon_1(p')) \left(1 - \cos((\mathbf{p}, \mathbf{p}'))\right) \frac{sd^2 p'}{(2\pi\hbar)^2} . \quad (7)$$

Substituting Eqs. (A12) and (A10) into Eq. (7), we get

$$\tau_1(p) = \frac{\varepsilon_1(p)}{\xi(p)} \tau_p(p) , \quad (8)$$

where $\tau_p(p)$ is the polariton relaxation time in the normal phase given by

$$\frac{1}{\tau_p(p)} = \frac{2\pi}{\hbar} \int |V(\mathbf{p}, \mathbf{p}')|^2 X_{\mathbf{p}}^2 X_{\mathbf{p}'}^2 \delta(\varepsilon_0(p) - \varepsilon_0(p')) \left(1 - \cos((\mathbf{p}, \mathbf{p}'))\right) \frac{sd^2 p'}{(2\pi\hbar)^2} . \quad (9)$$

Hence, we obtain $\tau_p(p) = \tau_n(p)/X_{\mathbf{p}}^4$, where $\tau_n(p)$ is the exciton relaxation time in the normal phase given by

$$\frac{1}{\tau_n(p)} = \frac{2\pi}{\hbar} \int |V(\mathbf{p}, \mathbf{p}')|^2 \delta(\varepsilon_{ex}(p) - \varepsilon_{ex}(p')) \left(1 - \cos((\mathbf{p}, \mathbf{p}'))\right) \frac{sd^2 p'}{(2\pi\hbar)^2} , \quad (10)$$

where $\varepsilon_{ex}(p)$ is the energy spectrum of the excitons. In the case of excitations with a small momentum, where $\varepsilon_0(p) \ll \mu$, the dispersion law of the excitations has an acoustic form: $\varepsilon_1(p) = c_s p$, and from Eq. (8) we have $\tau_1(p) = (p/(M_p c_s)) \tau_p(p)$. Therefore, in the presence of the superfluidity the relaxation time of polariton excitations $\tau_1(p)$ can be obtained from the exciton normal phase relaxation time $\tau_n(p)$ as $\tau_1(p) = (X_{\mathbf{p}}^4 p/(M_p c_s)) \tau_n(p)$. Without the superfluidity, at $\sigma(p_1, p'_1) = X_{\mathbf{p}_1} X_{\mathbf{p}'_1}$ we have $\tau_1(p) = X_{\mathbf{p}}^4 \tau_n(p)$. The exciton normal phase relaxation time $\tau_n(p)$ can be approximated by its average value $\bar{\tau}_n = \langle \tau_n(p) \rangle$, which can be obtained from the exciton mobility $\tilde{\mu}_{ex} = e \bar{\tau}_n / M$. The exciton mobility $\tilde{\mu}_{ex}$ is presented in Figs. 1 and 2 in Ref. [43].

IV. THE DRAG COEFFICIENTS

We introduce the drag coefficients λ_p and λ_{ex} for electrons in the 2DEG dragged by the moving polaritons and excitons, respectively. For the case when the electric field is applied to the system of electrons in the QW we introduce the drag coefficients γ_p and γ_{ex} , respectively, for polaritons and excitons dragged by the electron current. In the two-layer system there are a current of electrons and flow of polaritons or excitons. The flow of polaritons or excitons is $\mathbf{i}_i = n_i \mathbf{v}_i$, where n_i and v_i are density and average velocity, and the index i is defined as $i = ex$ for excitons and $i = p$ for polaritons. The electron current $\mathbf{j} = -en_{el} \mathbf{v}_{el}$, where n_{el} is the density and \mathbf{v}_{el} is the average velocity of electrons in the electron layer. These currents can be expressed in terms of the density gradient ∇n_i in the polariton

or exciton subsystem, drag coefficients λ_i and γ_i , and external electric field \mathbf{E} applied to the 2DEG by the following matrix expression:

$$\begin{pmatrix} \mathbf{i}_i \\ \mathbf{j} \end{pmatrix} = \begin{pmatrix} -D_i & \gamma_i \\ \lambda_i & en_{el}D_e \end{pmatrix} \cdot \begin{pmatrix} \nabla n_i \\ \mathbf{E} \end{pmatrix}, \quad (11)$$

where D_i is the polariton or exciton diffusion coefficient and D_e is the mobility coefficient of the electrons. Only normal component in the polariton subsystem is dragged by the electron current, while the superfluid component is not dragged. Thus, the appearance of the polariton superfluidity can be detected by the electron-polariton drag effect.

A. The drag of the polariton quasiparticles by the electron current.

Let us find the drag coefficient γ_p by following the procedure applied in Ref. [21] for the derivation of the drag coefficient related to the drag of the quasiparticles in the polariton subsystem by the electrons. We obtain the drag coefficient γ_p from the expansion of the polariton flow \mathbf{i}_p in the first order with respect to \mathbf{E} . The expression for the polariton flow \mathbf{i}_p is given by

$$\mathbf{i}_p = -\frac{1}{M_p} \int \mathbf{p}_1 n(\mathbf{p}_1) \frac{sd^2 p_1}{(2\pi\hbar)^2}, \quad (12)$$

where \mathbf{p}_1 is the polariton momentum, $s = 4$ is the degeneracy factor for polaritons, and $n(\mathbf{p}_1)$ is the distribution function of the quasiparticle excitations in the polariton subsystem which can be found using kinetic equations. The kinetic equations for distribution function of the quasiparticles are represented in Appendix B.

Using the distribution function of the quasiparticles from Appendix B, we obtain the polariton flow \mathbf{i}_p in the first order with respect to external electric field \mathbf{E} , and find the drag coefficient γ_p . As a result, we obtain for γ_p :

$$\begin{aligned} \gamma_p &= \frac{\pi}{2\hbar} \frac{e}{M_p m_e k_B T} \int \frac{sd^2 q}{(2\pi\hbar)^2} W_{eff}^2(q, D) \\ &\times \int_0^\infty \frac{\tilde{\Phi}(\mathbf{q}, \xi) \Psi(\mathbf{q}, \xi)}{\sinh^2(\xi/(2k_B T))} d\xi, \end{aligned} \quad (13)$$

where

$$\begin{aligned} \tilde{\Phi}(\mathbf{q}, \xi) &= \int \sigma^2(p_1, |\mathbf{p}_1 + \mathbf{q}|) [n_0(\varepsilon_1(\mathbf{p}_1 + \mathbf{q})) - n_0(\varepsilon_1(p_1))] \\ &\times (\tau_p(|\mathbf{p}_1 + \mathbf{q}|)(\mathbf{p}_1 + \mathbf{q}) - \tau_p(p_1)\mathbf{p}_1) \delta(\varepsilon_1(p_1) - \varepsilon_1(|\mathbf{p}_1 + \mathbf{q}|) + \xi) \frac{sd^2 p_1}{(2\pi\hbar)^2} \\ &\approx \frac{2s}{\pi\hbar^2 c_s^2} \mathbf{q} \int \left(\sigma^2(p_1, |\mathbf{p}_1 + \mathbf{q}|) \frac{\partial n_0(\varepsilon_1)}{\partial \varepsilon_1} \right) \Big|_{\varepsilon_1=\mu} (\varepsilon_1(|\mathbf{p}_1 + \mathbf{q}|) - \varepsilon_1(p_1)) \bar{\tau}_n \delta(\varepsilon_1(p_1) - \varepsilon_1(|\mathbf{p}_1 + \mathbf{q}|) + \xi) \varepsilon_1 d\varepsilon_1 \\ &\approx -\frac{s\mu\bar{\tau}_n}{2\pi\hbar^2 c_s^2 k_B T} \frac{\exp[\mu/(k_B T)]}{(\exp[\mu/(k_B T)] - 1)^2} \xi \mathbf{q}, \end{aligned} \quad (14)$$

and

$$\begin{aligned} \Psi(\mathbf{q}, \xi) &= \int [f_0(\varepsilon_2(|\mathbf{p}_2 + \mathbf{q}|)) - f_0(\varepsilon_2(p_2))] (\tau_2(|\mathbf{p}_2 + \mathbf{q}|)(\mathbf{p}_2 + \mathbf{q}) - \tau_2(p_2)\mathbf{p}_2) \\ &\times \delta(\varepsilon_2(p_2) - \varepsilon_2(|\mathbf{p}_2 + \mathbf{q}|) + \xi) \frac{2 d^2 p_2}{(2\pi\hbar)^2} \\ &\approx \frac{m_e}{\pi\hbar^2} \mathbf{q} \int \left(\frac{\partial f_0(\varepsilon_2)}{\partial \varepsilon_2} \right) \Big|_{\varepsilon_2=\varepsilon_F} (\varepsilon_2(|\mathbf{p}_2 + \mathbf{q}|) - \varepsilon_2(p_2)) \bar{\tau}_2 \delta(\varepsilon_2(p_2) - \varepsilon_2(|\mathbf{p}_2 + \mathbf{q}|) + \xi) d\varepsilon_2 \\ &= -\frac{m_e \bar{\tau}_2}{4\pi\hbar^2 k_B T} \xi \mathbf{q}. \end{aligned} \quad (15)$$

Assuming the system to be close to the equilibrium and considering $\nabla\mu_{qp}(\mathbf{r})$ to be very small, we put $\mu_{qp}(\mathbf{r}) = 0$ in Eqs. (13)- (15).

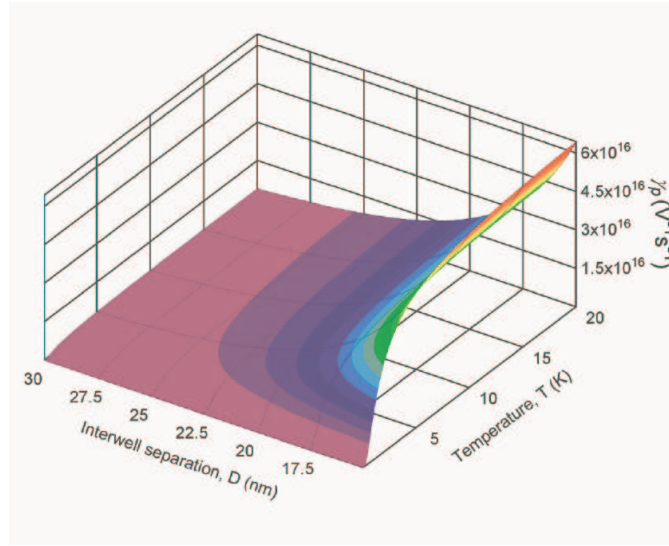


FIG. 2: (Color online) The drag coefficient γ_p in $\text{V}^{-1}\text{s}^{-1}$ in the system of superfluid microcavity polaritons and electrons as a function of temperature T in K and interwell separation D in nm. The polariton density $n_p = 10^{10} \text{ cm}^{-2}$. We used the parameters for the GaAs/GaAsAl quantum wells: $m_e = 0.07m_0$, $m_h = 0.15m_0$, $M = 0.24m_0$, $\epsilon = 13$.

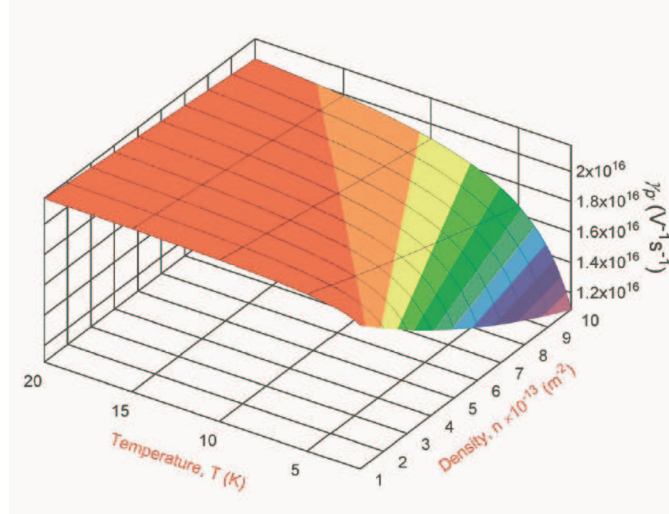


FIG. 3: (Color online) The drag coefficient γ_{pe} in $\text{V}^{-1}\text{s}^{-1}$ in the system of superfluid microcavity polaritons and electrons as a function of temperature T in K and polariton density n_p in m^{-2} . The interwell separation $D = 20 \text{ nm}$. We used the parameters for the GaAs/GaAsAl quantum wells: $m_e = 0.07m_0$, $m_h = 0.15m_0$, $M = 0.24m_0$, $\epsilon = 13$.

Let us mention that the typical interwell distances used in the drag experiment in Ref. [4] are 17.5 nm and 22.5 nm. The interwell distances used in the experiment in Ref. [18] are 20 nm and 30 nm. In our calculations we used the same interwell distances that used in the drag experiments [4, 18], namely 17.5 nm, 20 nm, 22.5 nm and 30 nm. Figs. 2 and 3 present results of calculations for the drag coefficient γ_p . The drag coefficient γ_p as a function of temperature T and interwell separation D is shown in Fig. 2. The drag coefficient γ_p as a function of temperature T and polariton density n_p is presented in Fig. 3. We can conclude that the drag coefficient γ_p exponentially decreases with the exciton density n , exponentially increases with the temperature T and decreases with the interwell separation D .

Let us mention that in the GaAs quantum wells used in Ref. [33] the Rabi splitting is 13 meV ($\sim 150 \text{ K}$). Since the derivation γ_p resulting in Eq. (13) implies low temperatures $k_B T \ll \mu$ and $k_B T \ll \hbar\Omega_R$, we can apply Eq. (13) for the temperatures below $\sim 20 \text{ K}$.

Note that Eq. (13) was obtained by using the regular Bogoliubov approximation for the weakly-interacting Bose gas with no dissipation. If the exciton-polaritons have a finite life-time in the cavity, the Bogoliubov dispersion is modified at small wave vectors [44]. This modification should affect the drag. However, we consider the small relative deviation from the threshold pumping intensity, when the pumping maintains the exact balance of amplification and losses.

According to Eq. (6) in Ref. [44], at small relative deviation from the threshold pumping intensity, the spectrum of collective excitations in the system of exciton polaritons corresponds to the regular Bogoliubov approximation. We assume that the leakage of the photons from the microcavity is very small, and the system can be considered in the quasi-equilibrium.

B. The drag of the electrons by the flow of polariton quasiparticles.

Let us find the drag coefficient λ_p related to the drag of the electrons by the quasiparticles in the polariton subsystem. We obtain the drag coefficient λ_p from the expansion of the electron current \mathbf{j} in the first order with respect to ∇n_{qp} , where n_{qp} is the density of quasiparticles contributing to the normal component of the polariton subsystem. The expression for the electron current \mathbf{j} is given by

$$\mathbf{j} = -\frac{2e}{m_e} \int \mathbf{p}_2 f(\mathbf{p}_2) \frac{d^2 p}{(2\pi\hbar)^2}, \quad (16)$$

where \mathbf{p}_2 is the electron momentum. and $f(\mathbf{p}_2)$ is the electron distribution function represented by Eq. (B4). We can find $g_2(\mathbf{p}_2)$ in Eq. (B4) by solving the kinetic equations Eqs. (B2)-(B3).

Substituting Eq. (B7) into Eq. (B6), we expand the kinetic equations (B2) and (B3) in the first order with respect to $\nabla \mu_{qp}$, we find the functions g_1 and g_2 . Therefore, we can obtain the electron current \mathbf{j} in the first order with respect to ∇n_{qp} , and find the drag coefficient λ_p . As a result, we obtain λ_p :

$$\begin{aligned} \lambda_p &= \frac{\pi}{2\hbar} \frac{e}{M_p m_e k_B T} \left(\frac{\partial n_{qp}}{\partial \mu_{qp}} \right)^{-1} \int \frac{sd^2 q}{(2\pi\hbar)^2} W_{eff}^2(q, D) \\ &\times \int_0^\infty \frac{\tilde{\Phi}(\mathbf{q}, \xi) \Psi(\mathbf{q}, \xi)}{\sinh^2(\xi/(2k_B T))} d\xi, \end{aligned} \quad (17)$$

where $\tilde{\Phi}(\mathbf{q}, \xi)$ and $\Psi(\mathbf{q}, \xi)$ are given by Eqs. (14) and (15), correspondingly.

Applying for the density of the quasiparticles n_{qp}

$$n_{qp} = \int \frac{1}{\exp([\varepsilon_1(\mathbf{p}_1) - \mu_{qp}(\mathbf{r})]/(k_B T)) - 1} \frac{sd^2 p_1}{(2\pi\hbar)^2}, \quad (18)$$

we obtain $(\partial n_{qp})/(\partial \mu_{qp})$:

$$\begin{aligned} \left(\frac{\partial n_{qp}}{\partial \mu_{qp}} \right) \Big|_{\mu_{qp}=0} &= \int \frac{\partial}{\partial \mu_{qp}} \Big|_{\mu_{qp}=0} \left(\frac{1}{\exp([\varepsilon_1(\mathbf{p}_1) - \mu_{qp}(\mathbf{r})]/(k_B T)) - 1} \right) \frac{sd^2 p_1}{(2\pi\hbar)^2} \\ &= \frac{1}{k_B T} \int \frac{\exp[\varepsilon_1(\mathbf{p}_1)/(k_B T)]}{(\exp[\varepsilon_1(\mathbf{p}_1)/(k_B T)] - 1)^2} \frac{sd^2 p_1}{(2\pi\hbar)^2} = \frac{sk_B T}{2\pi\hbar^2 c_s^2} \int_0^\infty \frac{e^x x dx}{(e^x - 1)^2}, \end{aligned} \quad (19)$$

where we use $\varepsilon_1(\mathbf{p}_1) = c_s p_1$ and $x = c_s p_1 / k_B T$ for the small momenta. The integral in the r.h.s. of Eq. (19) diverges at $p \rightarrow 0$. Therefore, we have $(\partial n_{qp})/(\partial \mu_{qp}) \rightarrow \infty$, which results in $\lambda_p = 0$ according to Eq. (17). This result comes from the assumption that for the very dilute Bose gas of polaritons we took into account only the sound region of the collective excitation spectrum at small momenta, and neglect almost not occupied regions with the quadratic spectrum at large momenta and crossover dependence of the spectrum at the intermediate momenta. Our approximation results in suppressed λ_p . Therefore, in the presence of the superfluidity of polaritons, the polaritons moving due to their density gradient almost do not drag electrons and there is suppressed electron current induced by the polaritons. Hence, the suppression of the dragged electric current in the electron QW can indicate the superfluidity of the polaritons.

V. THE DRAG EFFECTS IN THE EXCITON-ELECTRON SYSTEM AT HIGH TEMPERATURES

For high temperature, $k_B T \gtrsim \hbar \Omega_R$, the majority of polaritons occupy the upper polariton branch, where the upper polariton mass is very close to the mass of exciton M_{ex} . So at high temperature the polaritons are replaced by the gas of excitons [39].

Without the superfluidity in the definitions of the drag coefficients presented by Eq. (11) it should be substituted the exciton density n_{ex} instead of the quasiparticle density n_{qp} , exciton mass M_{ex} instead of polariton mass M_p , and

the chemical potential of excitons μ_{ex} instead of the chemical potential of the quasiparticles μ_{qp} . The drag coefficients γ_{ex} for the exciton system without the superfluidity can be obtained from Eqs. (13)- (15) by substituting $\sigma(p_1, p'_1) = 1$, $\varepsilon_1(p_1) = p_1^2/(2M_{ex})$, and $n_0(\mathbf{p}_1) = \left(\exp([\varepsilon_1(\mathbf{p}_1) - \mu_{ex}^{(0)}(\mathbf{r})]/(k_B T)) - 1\right)^{-1}$ is the Bose-Einstein distribution function of the excitons at the equilibrium, where $\mu_{ex}^{(0)}$ is the chemical potential of the excitons in the equilibrium determined by the polariton density n_{ex} :

$$n_{ex} = \int \frac{1}{\exp([\varepsilon_1(\mathbf{p}_1) - \mu_{ex}^{(0)}]/(k_B T)) - 1} \frac{sd^2 p_1}{(2\pi\hbar)^2} . \quad (20)$$

From Eq. (20) we obtain the $\mu_{ex}^{(0)}$:

$$\mu_{ex}^{(0)} = k_B T \log \left[1 - \exp \left[-\frac{2\pi\hbar^2 n}{sM_{ex}k_B T} \right] \right] . \quad (21)$$

We get from Eq. (20):

$$n_{ex} = -\frac{sM_{ex}k_B T}{2\pi\hbar^2} \left(1 - \exp \left[\frac{\mu_{ex}(\mathbf{r})}{k_B T} \right] \right) . \quad (22)$$

From Eq. (22) we find:

$$\left(\frac{\partial n_{ex}}{\partial \mu_{ex}} \right) \Big|_{\mu_{ex}=\mu_{ex}^{(0)}} = \frac{sM_{ex}}{2\pi\hbar^2} \left[1 - \exp \left[-\frac{2\pi\hbar^2 n}{sM_{ex}k_B T} \right] \right] . \quad (23)$$

The coefficient λ_{ex} for the high temperature range is given by

$$\begin{aligned} \lambda_{ex} = & \left(\frac{\partial n_{ex}}{\partial \mu_{ex}} \Big|_{\mu_{ex}=\mu_{ex}^{(0)}} \right)^{-1} \frac{\pi}{2\hbar} \frac{e}{M_{ex}m_e k_B T} \int \frac{sd^2 q}{(2\pi\hbar)^2} W_{eff}^2(q, D) \\ & \times \int_0^\infty \frac{\tilde{\Phi}(\mathbf{q}, \xi) \Psi(\mathbf{q}, \xi)}{\sinh^2(\xi/(2k_B T))} d\xi , \end{aligned} \quad (24)$$

where $\tilde{\Phi}(\mathbf{q}, \xi)$ and $\Psi(\mathbf{q}, \xi)$ are defined by the substituting ex excitonic parameters described above to Eqs. (14) and (15), correspondingly.

The results of the calculations of the drag coefficients at high temperature are presented in Figs. 4, 5 and 6. The drag coefficient γ_{ex} as a function of temperature and exciton density is shown in Fig. 4, while the drag coefficient γ_{ex} as a function of temperature and interwell separation is shown in Fig. 5. The drag coefficient λ_{ex} at the high temperature range for electrons dragged by moving excitons as a function of temperature and interwell separation is presented in Fig. 6. Based on the results of calculations we can conclude that the drag coefficients γ_{ex} and λ_{ex} decrease with the exciton density, increase with the temperature and decrease with the interwell separation.

VI. PROPOSED EXPERIMENTS

We propose the following experiments relevant to electron-polariton and electron-exciton drag effect. Let the screen with two diaphragms covers the quantum well embedded into a semiconductor microcavity. The laser pumping through one diaphragm generates excitons forming the polaritons by coupling to the cavity photons. At low temperature regime the absence of the electron current will indicate the superfluidity phase of the polaritons, while at high temperature regime the existence of the electron current will indicate the drag of electrons by the moving excitons. Therefore, the drag coefficient λ_{ex} allows to estimate the dragged electron current. When the electron current is induced by the external electric field applied to the electron QW, the photoluminescence spectrum can be measured in the other diaphragm. At low temperature, the difference between the photoluminescence spectrum of polaritons decay with the electric field applied to the electrons and without the electric field will indicate that the polaritons moved to the other place of the QW due to the drag by the electron current. The photoluminescence spectrum in the other diaphragm without the electric field is caused only by the diffusion of the polaritons. Only the normal component of the polariton subsystem will move to the other diaphragm, while the superfluid component is not affected by the electrons. It seems like electrons move the photons that coupled with excitons. At high temperature regime the

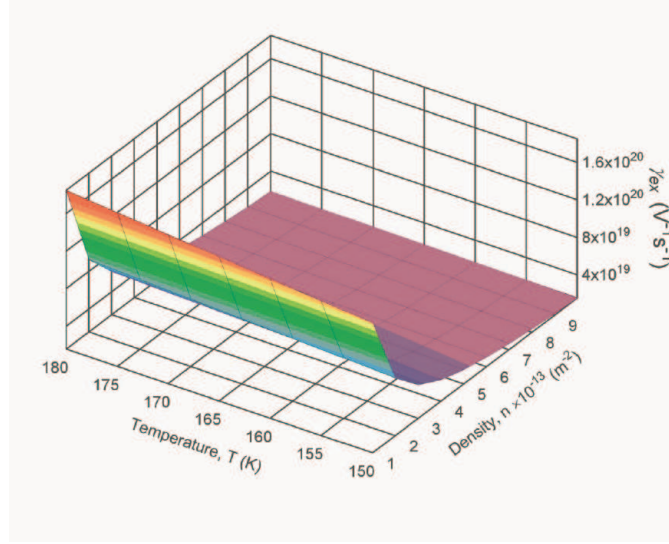


FIG. 4: (Color online) The drag coefficient γ_{exe} in $\text{V}^{-1}\text{s}^{-1}$ in the system of spatially separated excitons and electrons without superfluidity as a function of temperature T in K and exciton density n_{ex} in m^{-2} . The interwell separation is $D = 20$ nm. We used the parameters for the GaAs/GaAsAl quantum wells: $m_e = 0.07m_0$, $m_h = 0.15m_0$, $M = 0.24m_0$, $\epsilon = 13$.

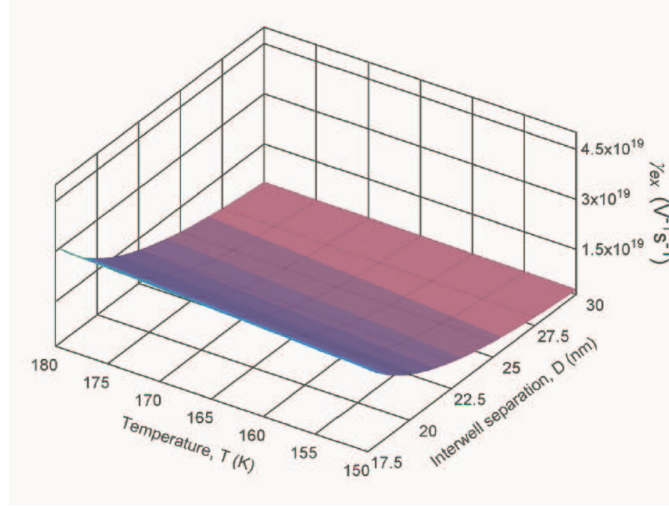


FIG. 5: (Color online) The drag coefficient γ_{exe} in $\text{V}^{-1}\text{s}^{-1}$ in the system of spatially separated excitons and electrons without superfluidity as a function of temperature T in K and interwell separation D in nm. The exciton density $n_{ex} = 10^{10} \text{ cm}^{-2}$. We used the parameters for the GaAs/GaAsAl quantum wells: $m_e = 0.07m_0$, $m_h = 0.15m_0$, $M = 0.24m_0$, $\epsilon = 13$.

photoluminescence spectrum of the electron-hole recombination indicates that the excitons moved to the other place of the QW due to the drag by the electron current.

The other suggested experiment is based on the observation of the angular distribution of the photons escaping the optical microcavity. At low temperature regime we propose to create the uniform distribution of polaritons by the laser pumping within the microcavity. Therefore, $\nabla n_p = 0$ and there is no polariton flow. In the absence of polariton flow the average angle of the photons escaping the optical microcavity and the perpendicular to the microcavity is $\bar{\alpha} = 0$, because the angular distribution is symmetrical [32]. Let us induce the electron current by applying the electric field \mathbf{E} and analyze the photon angular distribution in the presence of the nonzero current of polaritons along the quantum well parallel to the cavity dragged by the electron current due to the drag effect. If $\nabla n_p = 0$ the polariton flow is $\mathbf{i}_p = \gamma_p \mathbf{E}$, and according to the definition of polariton flow we have $\mathbf{v}_p = \gamma_p \mathbf{E} / n_p$. Therefore, we can obtain the average component of the polariton momentum in the direction parallel to the Bragg mirrors of the microcavity: $\overline{p_{||}} = M_p v_p = M_p \gamma_p E / n_p$. Since the perpendicular to the Bragg mirrors component of the polariton momentum is given by $p_{\perp} = \hbar \pi / L_C$ [25], we obtain for the average tangent of the angle between the path of the escaping photon

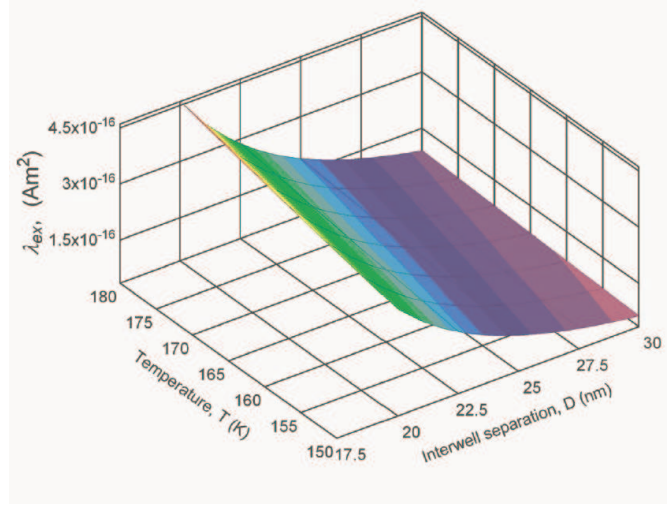


FIG. 6: (Color online) The drag coefficient λ_{ex} in A m^2 in the system of spatially separated excitons and electrons without superfluidity as a function of temperature T in K and interwell separation D in nm. The exciton density $n_{ex} = 10^{10} \text{ cm}^{-2}$. We used the parameters for the GaAs/GaAsAl quantum wells: $m_e = 0.07m_0$, $m_h = 0.15m_0$, $M = 0.24m_0$, $\epsilon = 13$.

and the perpendicular to the microcavity:

$$\overline{\tan \alpha} = \frac{\overline{p_{||}}}{p_{\perp}} = \frac{\gamma_p M_p L_C E}{\hbar \pi n_p}. \quad (25)$$

Note that only normal component of the polariton subsystem will contribute to the drag coefficient γ_p and therefore to $\overline{\tan \alpha}$. There will be two peaks of the escaping photons: one at $\overline{\tan \alpha} \neq 0$ corresponds to the moving (dragged) normal component, and the other one at $\overline{\tan \alpha} = 0$ corresponds to the superfluid component. Note that the analysis of the angular distributions of the photons escaping the optical microcavity has been used in the experiments [40, 41]. The manifestation of polariton drag effect through the change of the angular distribution of the photons escaping the optical microcavity is shown in Fig. 7. Only quasiparticles in polariton system are dragged by electrons.

Let us make estimations of the parameters for the drag effects. At the temperature $T = 4 \text{ K}$ the experiment [32, 33] shows that the polariton lifetime $\tau = 10 \text{ ps}$, and the polariton diffusion path is $l = 20 \mu\text{m}$. The corresponding average polariton velocity is $v_p = l/\tau = 2 \times 10^6 \text{ m/s}$. Since $E = n_p v_p / \gamma_p$, we can estimate the electric field E corresponding to such drag effect. For $n_p = 10^{10} \text{ cm}^{-2}$ and $T = 4 \text{ K}$ for the interwell separation $D = 17.5 \text{ nm}$ $\gamma_p = 2.64 \times 10^{16} \text{ V}^{-1}\text{s}^{-1}$, the corresponding electric field is $E = 3.8 \times 10^3 \text{ V/m}$ which corresponds to the applied voltage $V = 3.8 \times 10^{-3} \text{ V}$ at the size of the system $d = 1 \mu\text{m}$. Using $M_p = 7 \times 10^{-5} \times m_e$, the length of the microcavity $L_C = 2 \mu\text{m}$ [32, 33], and the estimated γ_p and E in (25), we obtain for the average tangent of the angle α : $\overline{\tan \alpha} = 0.385$ and $\tan^{-1}(\overline{\tan \alpha}) = 21^\circ$.

VII. DISCUSSION AND CONCLUSIONS

Let us mention that we calculated the drag coefficients as the linear responses of the equilibrium system, and, therefore, our formulas for the drag coefficients are expressed in terms of the parameters of the system at the equilibrium.

The other approximation used in our approach is that we considered the polariton-electron drag only due to exciton-electron interaction, neglecting the contribution coming from the cavity photon-electron interaction. There are several processes caused by the photon-electron interaction. The magnitude of the direct cavity photon-electron static scattering processes are the second order with respect to the fine-structure constant, and, therefore, their contributions are much smaller than the contribution caused by the exciton-electron interaction. The frequency of the plasmon excitations in 2DEG are proportional to $k^{1/2}$, where k is the wave vector, and the characteristic k is by the order of magnitude of $2m_e e^2 / (\hbar^2 \epsilon)$ due to the screening [42]. Therefore, the characteristic frequencies corresponding to the plasmon excitations in 2DEG are in THz range, while the characteristic frequencies of the cavity photons are in the optical range by the magnitude of 10^{15} Hz . Thus the plasmon excitations in 2DEG are not in the resonance with the cavity photons, while we consider polaritons formed by the excitons and cavity photons in the resonance. The electron intersubband transitions corresponding to the transitions between the different levels of the quantization in the QW are also not in the resonance with the cavity photons, because the characteristic width of the QW d managing the frequency of these intersubband transitions is much smaller than the size of microcavity L_C managing the frequency

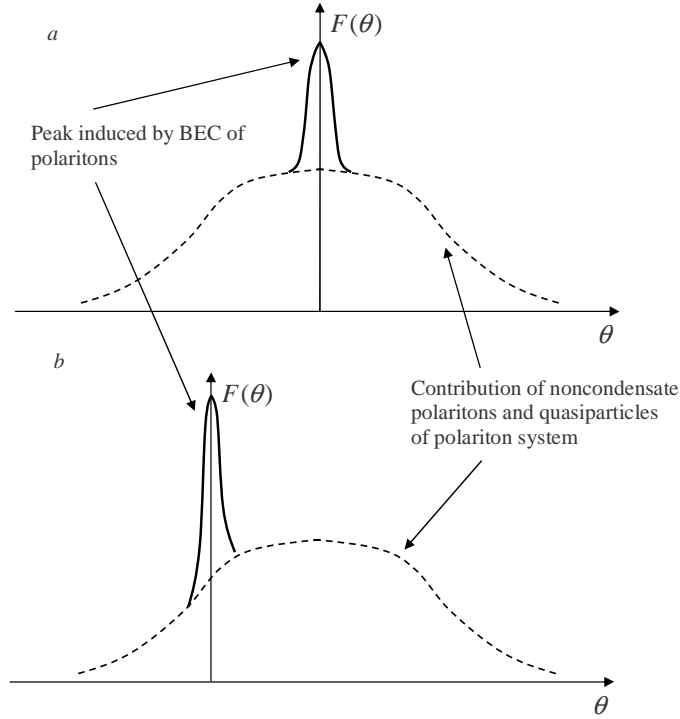


FIG. 7: Proposed experiment: the manifestation of polariton drag effect through change of the angular distribution of the photons escaping the optical microcavity. a. The angular distribution of the photons escaping the optical microcavity without drag. b. The angular distribution of the photons escaping the optical microcavity in the presence of the drag effect. Redistribution of the contribution of noncondensate polaritons.

of the cavity photons. The frequencies of the electron interband transitions in the semiconductor corresponding to the range in the continuous electron spectrum, where there is no discrete levels, also cannot be in the resonance with the microcavity photons. The screening of the microcavity photon spectrum by the 2DEG causes shift in the spectrum of microcavity photons, and therefore, the frequencies corresponding to the exciton-photon resonance related to the formation of the polariton are shifted. However, the formalism and procedure for the calculation of the polariton-electron drag coefficients will be the same as presented in this Paper. The rigorous analysis of the electron-microcavity photon drag effects is the very interesting direction for the future research.

We can conclude that for systems of spatially separated interacting quasiparticles is the possibility of controlling the motion of the quasiparticles of one subsystem by altering the parameters of state of the quasiparticles in the other subsystem, for example, controlling the flow of polaritons or exciton using a current of electrons. At low temperatures the electron current dragged by the polariton flow is strongly suppressed and hence, the absence of the electron current indicates the superfluidity of polaritons. However, the polariton flow can be dragged by the electrons, and, therefore, there is a transport of photons along the microcavity, which decreases with rise of the superfluid component and can be observed through the change in angular distribution of photons discussed above. At high temperatures, from one hand, the existence of the electric current in the electron QW indicates the exciton flow in the other QW, and from the other hand, the electron current in one QW induces the exciton flow in the other QW via the drag of excitons by the electrons. The obtained drag coefficients allow calculate the corresponding currents. According to our calculations, the low temperature drag coefficient γ_p and the high temperature drag coefficients γ_{ex} and λ_{ex} decrease with the exciton density, increase with the temperature and decrease with the interwell separation. The suggested experiments allow to observe the analyzed drag effects. We suggested the experiment for the observation of the distributions of the angles α between the path of the photons escaping the microcavity and the perpendicular to the Bragg mirrors. The average tangent of the angle $\tan \alpha$ between the path of the escaping photon and the perpendicular to the microcavity is proportional to the drag coefficient γ_p and the electric field \mathbf{E} applied to the electrons.

Acknowledgments

O. L. B., R. Ya. K. were supported by PSC CUNY grant 63443-00 41 and grant 62136-00 40; Yu. E. L. and A. A. K. were supported by RFBR and RAS programs.

Appendix A: The quasiparticle operators representation for the Hamiltonian of the exciton-electron interaction

Let us express the exciton operators in terms of polariton operators. The exciton and photon operators are defined as [45]

$$\hat{a}_{\mathbf{p}} = X_p \hat{l}_{\mathbf{p}} - C_p \hat{w}_{\mathbf{p}} , \quad \hat{d}_{\mathbf{p}} = C_p \hat{l}_{\mathbf{p}} + X_p \hat{w}_{\mathbf{p}} , \quad (\text{A1})$$

where $\hat{l}_{\mathbf{p}}$ and $\hat{w}_{\mathbf{p}}$ are lower and upper polariton Bose operators, respectively, X_p and C_p are

$$X_p = \left(1 + \left(\frac{\hbar\Omega_R}{\varepsilon_{LP}(p) - \varepsilon_{ph}(p)} \right) \right)^{-1/2} , \quad C_p = - \left(1 + \left(\frac{\varepsilon_{LP}(p) - \varepsilon_{ph}(p)}{\hbar\Omega_R} \right) \right)^{-1/2} , \quad (\text{A2})$$

and the energy spectra of the low/upper polaritons are

$$\begin{aligned} \varepsilon_{LP/UP}(p) &= \frac{\varepsilon_{ph}(p) + \varepsilon_{ex}(p)}{2} \\ &\mp \frac{1}{2} \sqrt{(\varepsilon_{ph}(p) - \varepsilon_{ex}(p))^2 + 4|\hbar\Omega_R|^2} . \end{aligned} \quad (\text{A3})$$

Eq. (A3) implies a splitting between the upper and lower states of polaritons at $p = 0$ of $2\hbar\Omega_R$, known as the Rabi splitting. Let us also mention that $|X_p|^2$ and $|C_p|^2 = 1 - |X_p|^2$ represent the exciton and cavity photon fractions in the lower polariton.

In Eq. (A3), $\varepsilon_{ex}(p)$ is the energy dispersion of a single exciton in a quantum well given by

$$\varepsilon_{ex}(p) = E_{band} - E_{binding} + \tilde{\varepsilon}_0(p) , \quad (\text{A4})$$

where E_{band} is the band gap energy, $E_{binding} = \mu_0 e^4 / (\hbar^2 \epsilon)$ is the binding energy of a 2D exciton, and $\tilde{\varepsilon}_0(p) = p^2 / (2M_{ex})$, where $M_{ex} = m_e + m_h$ is the mass of an exciton. The cavity photon spectrum is given by

$$\varepsilon_{ph}(p) = (c/\tilde{n}) \sqrt{p^2 + \hbar^2 \pi^2 L_C^{-2}} . \quad (\text{A5})$$

In Eq. (A5), c is the speed of light, L_C is the length of the cavity, $\tilde{n} = \sqrt{\epsilon}$ is the effective refractive index. We assume the length of microcavity has the following form

$$L_C = \frac{\hbar\pi c}{\tilde{n}(E_{band} - \mathcal{E}_{binding})} . \quad (\text{A6})$$

So the photonic and excitonic branches start at the resonance at $p = 0$. This means that $\varepsilon_{ex}(p) = \varepsilon_{ph}(p)$ at $p = 0$ if L_C satisfies to Eq. (A6).

At small momenta $\alpha \equiv 1/2(M_{ex}^{-1} + (c/\tilde{n})L_C/\hbar\pi)p^2/|\hbar\Omega_R| \ll 1$, the single-particle lower polariton spectrum obtained by substitution of Eq. (A4) into Eq. (A3), in linear order with respect to the small parameters α , is

$$\varepsilon_0(p) \approx \frac{c}{\tilde{n}} \hbar\pi L_C^{-1} - |\hbar\Omega_R| + \frac{\gamma}{4} r^2 + \frac{1}{4} \left(M_{ex}^{-1} + \frac{cL_C}{\tilde{n}\hbar\pi} \right) p^2 = \frac{p^2}{2M_p} , \quad (\text{A7})$$

where M_p is the effective mass of polariton given by

$$M_p = 2 \left(M_{ex}^{-1} + \frac{cL_C}{\tilde{n}\hbar\pi} \right)^{-1} . \quad (\text{A8})$$

Substituting Eq. (A1) into Eq. (4), and taking into account only the lower polaritons corresponding to the lower energy, we obtain the Hamiltonian \hat{H}_{ex-el} expressed in terms of the lower polariton operators:

$$\hat{H}_{ex-el} = \frac{1}{A} \sum_{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}'_1, \mathbf{p}'_2} W_{eff}(|\mathbf{p}_1 - \mathbf{p}'_1|, D) X_{\mathbf{p}_1} X_{\mathbf{p}'_1} \hat{l}_{\mathbf{p}'_1}^\dagger \hat{c}_{\mathbf{p}'_2}^\dagger \hat{c}_{\mathbf{p}_2} \hat{l}_{\mathbf{p}_1} . \quad (\text{A9})$$

Now let us consider the two-layer polariton-electron system in the presence of the polariton superfluidity at $k_B T < \hbar\Omega_R$. At temperatures about the Rabi splitting $k_B T \gtrsim \hbar\Omega_R$, the upper polaritons states become filled, and the lower polaritons systems is replaced by the system of the upper polaritons, which are primarily excitons.

In the presence of the superfluidity we can obtain the Hamiltonian for the interaction of of quasiparticle excitations in a system of spatially separated polaritons and electrons by using the Bogoliubov unitary transformations [46]:

$$\begin{aligned} \hat{l}_{\mathbf{p}} &= u_p \hat{b}_{\mathbf{p}} + v_p \hat{b}_{-\mathbf{p}}^\dagger, & \hat{l}_{\mathbf{p}}^\dagger &= u_p \hat{b}_{\mathbf{p}}^\dagger + v_p \hat{b}_{-\mathbf{p}}, \\ u_p &= (1 - F_p^2)^{1/2}, & v_p &= F_p u_p, \\ F_p &= (\varepsilon_1(p) - \xi(p))/\mu, & \varepsilon_1(p) &= (\xi^2(p) - \mu^2)^{1/2}, \\ \xi(p) &= \varepsilon_0(p) + \mu, \end{aligned} \quad (\text{A10})$$

where $\hat{b}_{-\mathbf{p}}^\dagger$ and $\hat{b}_{-\mathbf{p}}$ are the creation and annihilation operators of the quasiparticle excitations, $\varepsilon_1(p)$ is the energy spectrum of the quasiparticle excitations in the polariton subsystem, and $\mu = M_p c_s^2$ is the polariton chemical potential in the Bogoliubov approximation [34], $c_s = (U_{eff}^{(0)} n_p / M_p)^{1/2}$ is the sound velocity in the polariton system, n_p is the 2D density of polaritons, $U_{eff}^{(0)} = 3e^2 a_0 / (2\epsilon)$ is the Fourier image of the polariton-polariton interaction [34]. Substituting $\hat{l}_{-\mathbf{p}}^\dagger$ and $\hat{l}_{-\mathbf{p}}$ from Eq. (A10) into Eq. (4), we obtain

$$\hat{H}_{ex-el} = \frac{1}{A} \sum_{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}'_1, \mathbf{p}'_2} W_{eff}(|\mathbf{p}_1 - \mathbf{p}'_1|, D) \sigma(p_1, p'_1) \hat{b}_{\mathbf{p}_1}^\dagger \hat{c}_{\mathbf{p}'_2}^\dagger \hat{c}_{\mathbf{p}_2} \hat{b}_{\mathbf{p}_1}, \quad (\text{A11})$$

where $\sigma(p_1, p'_1)$ in the presence of the superfluidity at $T < T_c$ is given by

$$\sigma(p_1, p'_1) = (u_{p_1} u_{p'_1} + v_{p_1} v_{p'_1}) X_{\mathbf{p}_1} X_{\mathbf{p}'_1}, \quad (\text{A12})$$

and $\sigma(p_1, p'_1) = X_{\mathbf{p}_1} X_{\mathbf{p}'_1}$ at $T > T_c$ without the superfluidity. Let us mention that at small momenta $\alpha \ll 1$ we have $|X_p|^2 \approx |C_p|^2 \approx 1/2$.

Appendix B: The kinetic equations for distribution function of the quasiparticles

The distribution function of the quasiparticle excitations in the polariton subsystem $n(\mathbf{p}_1)$ is represented in a form

$$n(\mathbf{p}_1) = n_0(\mathbf{p}_1) + n_0(\mathbf{p}_1)(1 + n_0(\mathbf{p}_1))g_1(\mathbf{p}_1), \quad (\text{B1})$$

where $n_0(\mathbf{p}_1) = (\exp([\varepsilon_1(\mathbf{p}_1) - \mu_{qp}(\mathbf{r})]/(k_B T)) - 1)^{-1}$ is the Bose-Einstein distribution function of the quasiparticles in the polariton subsystem at the equilibrium, $g_1(\mathbf{p}_1)$ is the contribution to the quasiparticle distribution function corresponding to the non-equilibrium correction due to the gradient of the quasiparticle chemical potential $\mu_{qp}(\mathbf{r})$ determined by the external conditions, k_B is the Boltzmann constant. We can find $g_1(\mathbf{p}_1)$ by solving the kinetic equations:

$$\frac{\partial n}{\partial \mathbf{r}} \cdot \frac{\partial \varepsilon_1(p)}{\partial \mathbf{p}} - \frac{\partial n}{\partial \mathbf{p}} \cdot \frac{\partial \varepsilon_1(p)}{\partial \mathbf{r}} = I_1(n) + I_{12}(n, f), \quad (\text{B2})$$

$$\frac{\partial f}{\partial \mathbf{r}} \cdot \mathbf{v} + \frac{\partial f}{\partial \mathbf{p}} \cdot \dot{\mathbf{p}} = I_2(f) + I_{21}(f, n), \quad (\text{B3})$$

where I_1 and I_2 are the collision integrals of the quasiparticles in the polariton subsystems and electrons with the impurities, I_{12} and I_{21} are the collision integrals of the quasiparticles with the electrons, and $f(\mathbf{p}_2)$ is the electron distribution function represented in a form

$$f(\mathbf{p}_2) = f_0(\mathbf{p}_2) + f_0(\mathbf{p}_2)(1 - f_0(\mathbf{p}_2))g_2(\mathbf{p}_2), \quad (\text{B4})$$

where $f_0(\mathbf{p}_2) = (\exp((\varepsilon_2(\mathbf{p}_2) - \varepsilon_F)/(k_B T)) + 1)^{-1}$ is the Fermi-Dirac electron distribution function in the equilibrium, T is the temperature, ε_F is the electron Fermi energy, $\varepsilon_2(\mathbf{p}_2) = p_2^2/(2m_e)$ is the electron energy spectrum, $g_2(\mathbf{p}_2)$ is the contribution to the electron distribution function corresponding to the non-equilibrium correction due to the external electric field \mathbf{E} .

We apply the τ approximation for $I_1(n)$ and $I_2(f)$:

$$I_1(n) = (n_0(\mathbf{p}_1) - n(\mathbf{p}_1))/\tau_1(\mathbf{p}_1), \quad I_2(f) = (f_0(\mathbf{p}_2) - f(\mathbf{p}_2))/\tau_2(\mathbf{p}_2), \quad (\text{B5})$$

where $\tau_1(\mathbf{p}_1)$ and $\tau_2(\mathbf{p}_2)$ are the relaxation times of the quasiparticles excitations in the polariton subsystem and electrons, respectively. According to Ref. [47], $\tau_2(\mathbf{p}_2) \approx \bar{\tau}_2$ can be approximated by the relaxation time at the Fermi surface, which can be determined from the electron mobility $\tilde{\mu}_e = e\bar{\tau}_2/m_e$. In a quantum well GaAs/AlGaAs $\tilde{\mu}_e$ is presented in Fig. 2 in Ref. [47].

Since the collision integral I_{12} is only a perturbation with respect to I_1 , we neglect it and assume $I_{12} = 0$. The collision integral I_{21} has the form [21]

$$I_{21}(g_2, g_1) = 2 \int w(\mathbf{p}_1, \mathbf{p}_2; \mathbf{p}'_1, \mathbf{p}'_2) n_0(\mathbf{p}_1) (1 + n_0(\mathbf{p}'_1)) f_0(\mathbf{p}_2) (1 + f_0(\mathbf{p}'_2)) \\ \times (g_1(\mathbf{p}'_1) + g_2(\mathbf{p}'_2) - g_1(\mathbf{p}_1) - g_2(\mathbf{p}_2)) \delta(\varepsilon_1(p_1) + \varepsilon_2(p_2) - \varepsilon_1(p'_1) - \varepsilon_2(p'_2)) \frac{sd^2 p'_1}{(2\pi\hbar)^2} \frac{\nu_e d^2 p'_2}{(2\pi\hbar)^2}, \quad (\text{B6})$$

where s is the level degeneracy (equal to 4 for excitons in GaAs quantum wells), $w(\mathbf{p}_1, \mathbf{p}_2; \mathbf{p}'_1, \mathbf{p}'_2)$ is the probability of a collision between a quasiparticle from the polariton subsystem and an electron, which can be obtained in the Born approximation as

$$w(\mathbf{p}_1, \mathbf{p}_2; \mathbf{p}'_1, \mathbf{p}'_2) = \frac{2\pi}{\hbar} |W_{eff}(q, D) \sigma(p_1, p'_1)|^2, \quad (\text{B7})$$

where $\mathbf{q} = \mathbf{p}'_1 - \mathbf{p}_1 = \mathbf{p}_2 - \mathbf{p}'_2$.

Substituting Eq. (B7) into Eq. (B6), and expanding the kinetic equations (B2) and (B3) in the first order with respect to $\nabla\mu_{qp}$, we find the functions g_1 and g_2 .

-
- [1] Yu. E. Lozovik and V. I. Yudson, JETP Lett. **22**, 26 (1975); JETP **44**, 389 (1976); Physica A **93**, 493 (1978).
 - [2] M. B. Pogrebinskii, Fiz. Tekh. Poluprovodn. **11**, 637 (1977) (Engl. Transl. Sov. Phys.-Semicond. **11**, 372 (1977)).
 - [3] P.J. Price, *Physica* **117B** & **118B**, 750 (1983).
 - [4] T. J. Gramila, J. P. Eisenstein, A. H. MacDonald, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **66**, 1216 (1991).
 - [5] U. Sivan, P. M. Solomon, and H. Shtrikman, Phys. Rev. Lett. **68**, 1196 (1992).
 - [6] T. J. Gramila, J. P. Eisenstein, A. H. MacDonald, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **47**, 12957 (1993).
 - [7] A-P. Jauho and H. Smith, Phys. Rev. B **47**, 4420 (1993).
 - [8] L. Zheng and A. H. MacDonald, Phys. Rev. B **48**, 8203 (1993).
 - [9] Yu. M. Sirenko and P. Vasilopoulos, Phys. Rev. B **46**, 1611 (1992).
 - [10] H. C. Tso, P. Vasilopoulos, F. M. Peeters, Phys. Rev. Lett. **68**, 2516 (1992).
 - [11] K. Flensberg, and BenYu-Kuang Hu, Phys. Rev. Lett. **73**, 3572 (1994).
 - [12] G. Vignale and A. H. MacDonald, Phys. Rev. Lett. **76**, 2786 (1996).
 - [13] B. Tanatar and A. K. Das, Phys. Rev. B **54**, 13827 (1996).
 - [14] W.-K. Tse and S. Das Sarma, Phys. Rev. B **75**, 045333 (2007).
 - [15] N. P. R. Hill, J. T. Nicholls, E. H. Linfield, M. Pepper, D. A. Ritchie, A. R. Hamilton, and G. A. C. Jones, J. Phys.: Condens. Matter **8**, L557 (1996).
 - [16] R. Pillarisetty, Hwayong Noh, D. C. Tsui, E. P. De Poortere, E. Tutuc, and M. Shayegan, Phys. Rev. Lett. **89** 016805 (2002).
 - [17] L. Tiemann, J. G. S. Lok, W. Dietsche, K. von Klitzing, K. Muraki, D. Schuh, and W. Wegscheider, Phys. Rev. B **77**, 033306 (2008).
 - [18] J. A. Seamons, C. P. Morath, J. L. Reno, and M. P. Lilly, Phys. Rev. Lett. **102**, 026804 (2009).
 - [19] J. M. Kosterlitz and D. J. Thouless, J. Phys. C **6**, 1181 (1973); D. R. Nelson and J. M. Kosterlitz, Phys. Rev. Lett. **39**, 1201 (1977).
 - [20] Yu. E. Lozovik and M. V. Nikitkov, JETP **84**, 612 (1997).
 - [21] Yu. E. Lozovik and M. V. Nikitkov, JETP **89**, 775 (1999).
 - [22] D. V. Kulakovskii and Yu. E. Lozovik, JETP **98**, 1205 (2004).
 - [23] Physica Status Solidi B **242**, 1 (2005), special issue of Physics of Semiconductor Microcavities, edited by B. Deveaud.
 - [24] A. Kavokin and G. Malpeuch, *Cavity Polaritons* (Elsevier, 2003).
 - [25] D. W. Snoke, *Solid State Physics. Essential Concepts*. (Addison-Wesley, 2008).
 - [26] V. Bagnato and D. Kleppner, Phys. Rev. A **44**, 7439 (1991).
 - [27] P. Nozières, in *Bose-Einstein Condensation*, A. Griffin, D. W. Snoke, and S. Stringari, Eds. (Cambridge Univ. Press, Cambridge, 1995), p.p. 15-30.
 - [28] J. Kasprzak, M. Richard, A. Baas, B. Deveaud, R. Andre, J.-Ph. Poizat, and Le Si Dang, Phys. Rev. Lett. **100**, 067402 (2008).
 - [29] S. Utsunomiya, L. Tian, G. Roumpos, C. W. Lai, N. Kumada, T. Fujisawa, M. Kuwata-Gonokami, A. Loeffler, S. Hoefling, A. Forchel and Y. Yamamoto, Nature Physics **4**, 700 (2008).

- [30] J. J. Baumberg, A. V. Kavokin, S. Christopoulos, A. J. D. Grundy, R. Butte, G. Christmann, D. D. Solnyshkov, G. Malpuech, G. Baldassarri Höger von Högersthal, E. Feltin, J.-F. Carlin, and N. Grandjean, *Phys. Rev. Lett.* **101**, 136409 (2008).
- [31] R. Balili, D. W. Snoke, L. Pfeiffer, and K. West, *Appl. Phys. Lett.* **88**, 031110 (2006).
- [32] R. Balili, V. Hartwell, D. W. Snoke, L. Pfeiffer and K. West, *Science* **316**, 1007 (2007).
- [33] R. Balili, B. Nelsen, D. W. Snoke, L. Pfeiffer, and K. West, *Phys. Rev. B* **79**, 075319 (2009).
- [34] O. L. Berman, Yu. E. Lozovik, and D. W. Snoke, *Phys. Rev. B* **77**, 155317 (2008).
- [35] J. Kasprzak, M. Richard, S. Kundermann, A. Baas, P. Jeambrun, J. M. J. Keeling, F. M. Marchetti, M. H. Szymanska, R. Andre, J. L. Staehli, V. Savona, P. B. Littlewood, B. Deveaud, and L. S. Dang, *Nature* **443**, 409 (2006).
- [36] O. L. Berman, R. Ya. Kezerashvili, and Yu. E. Lozovik, *Phys. Lett. A* **374**, 3681 (2010).
- [37] Yu. G. Rubo, A. V. Kavokin, and I. A. Shelykh, *Phys. Lett. A* **358**, 227 (2006).
- [38] T. C. H. Liew, Yu. G. Rubo, I. A. Shelykh, and A. V. Kavokin, *Phys. Rev. B* **77**, 125339 (2008).
- [39] P. Littlewood, *Science* **316**, 989 (2007).
- [40] A. Amo, J. Lefrère, S. Pigeon, C. Adrados, C. Ciuti, I. Carusotto, R. Houdrè, E. Giacobino, and A. Bramati, *Nature Phys.* **5**, 805 (2009).
- [41] A. Amo, D. Sanvitto, F. P. Laussy, D. Ballarini, E. del Valle, M. D. Martin, A. Lemaitre, J. Bloch, D. N. Krizhanovskii, M. S. Skolnick, C. Tejedor, and L. Vina, *Nature* **457**, 291 (2009).
- [42] T. Ando, A. B. Fowler, and F. Stern, *Rev. Mod. Phys.* **54**, 437 (1982).
- [43] P. K. Basu and P. Ray, *Phys. Rev. B* **44**, 1844 (1991).
- [44] M. Wouters and I. Carusotto, *Phys. Rev. Lett.* **99**, 140402 (2007).
- [45] C. Ciuti, P. Schwendimann, and A. Quattropani, *Semicond. Sci. Technol.* **18**, S279 (2003).
- [46] A. A. Abrikosov, L. P. Gorkov and I. E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics* (Prentice-Hall, Englewood Cliffs. N.J., 1963).
- [47] W. Walukiewicz, H. E. Ruda, J. Lagowski, and H. C. Gatos, *Phys. Rev. B* **30**, 4571 (1984).